GEOMETRY COURSEWARE DG AS A STEP FROM CLASSIC TO NEW EDUCATIONAL TECHNOLOGY

***Rakov S.A.**, Kharkov State Pedagogical University, rakov_s@ukr.net*
***Gorokh V.P.**, Kharkov State Pedagogical University, gorokh_v@ukr.net*

Nowadays the educational paradigm is rapidly changes enriching by the ideas of democratization and humanization, constructivism and implementing ICT. ICT play a very special role because of it is mainly not the aim but a tool for almost all these revolutionary changes. What is the key problem at this way? It is very important to look at the classic problems of the classic courses in a new way, transform routine exercises into learning research problems. This is the matter of the paper – new view at the classic geometry course, classic geometric problems. ICT, in particular an original package DG, gives a powerful handicap for that. All the discussed examples are excerpts from the courseware DG developed by the grant of the Ministry of education and science of Ukraine, which is in use now in the Ukrainian schools.

Introduction. The modern geometry course paradigm for school

Up to last time the educational tradition emphasizes the two main aims of the geometry course in school:

- Applied problems solving – develop as great as possible amount of skills for solving as wide as possible variety of applied (real life) problems;
- Developing of logical thinking – in the context of geometry the formal (logical) thinking in a form of axiomatic or deductive method was developed and only the geometry course in school follows it rather consistently.

The both of these two aims of the geometry course became the target for deep critic in the fall of 20th century. Really, the traditional applied problems are not very interesting for pupils (constructing and measuring on the earth). Mathematicians understood that there is no strict deducing at all and everybody chooses for himself/herself the proper level of strictness. As a consequence in many countries the geometry course was shrunk, in some of them – even avoided as an independent and self-sufficient course.

Nevertheless, “The God is excel but doesn’t cruel” - by the Albert Einstein’s words. The computer age has come and the traditional applied geometric problems became sound as a new ones – how to construct the picture at the computer display with traditional geometric primitives (constructing geometrical figures with a ruler and compass), how to animate them (geometrical transformations of figures), how to do it more efficient? In other words it was understood that in a matter of fact the Euclid geometry (constructing with a ruler and compass) is a brunch of the theory of algorithms (algorithms for specialized “geometric” automat) with analyzing all the aspects and properties of the algorithms. Thus the first aim of the geometry course do not “disappeared in the wind” but moreover became even more deep, interesting and important.

The new computer (information) age catalyzed the process of developing education practice. A lot of new metaphors were born, such as “learning during life”, “constructivism”, the mankind understood the main aim of school is to teach how to learn (more correctly – the school should create the conditions for self learning and self developing strategies of learning). “The role of a teacher nowadays was dramatically changed: from the sage at the stage to the guide by the side” as says the educational folklore. For good specialist it is not very important now to do something follow given algorithms. All the algorithmically formalized activities can and consequently should be carried out by computers. How to find suitable algorithm, how to choose the algorithm with the best characteristics, how to create new algorithm – these are more important skills for a modern specialist. The process of creating new algorithms in any subject area is not the algorithmically solvable problem – it is the really creative problem. Thus the school should to present “the real life” of the main branches of modern science in the process from approximate, “misty” knowledge to more strict and adequate knowledge. The spirit of modern school should be the spirit of projects, the spirit of explorations. The ICT gives a powerful handicap for the “constructive approach in education” for supporting constructing and explorations of computer models in any subject area. Computer models in geometry are not only important tool for studying the geometry itself in new way but as well a tool for developing geometric intuition. The second is crucially important because of geometrical intuition is the base for the cognitive modeling in any other subject area for majority of people. Thus the second aim of the school geometry course does not fail, moreover its role only spread from the logical thinking to the thinking.

But what kind the modern geometry course should be? It is a very complex and important question. In the light of two modern modified and improved up to date sounds of the main aims of the geometry course it should be (on our mind):

- Constructive - based on different kind explorations for conceptualizing, insights, reflecting the historical way of creating geometry and modern methods of professional math work;
- Real life - it should reflects as many as possible real life problems which are interesting and challenged for pupil and could be solved with geometry, in particular on computer with use of computer graphics;
- Computerized – closely integrated with specialized packages first of all for supporting conceptualizing, explorations, problem solving etc.

How to create such a new course? Such a course should be the result of an evolution and not only the new course will be created in this process but the new teachers and new pupils as well [1].

We think that one of the natural steps on this way should be the courseware develops mentioned above approaches at the base of current courses. The team consists of educators (scientists, teachers, programmers) prepared such a courseware “PMC DG” for

[1] Metaphorically not only the Chinese Civilization built the Great Wall but also the Chinese Civilization itself was built in the process of the Great Wall constructing.

supporting of a current geometry course based on the textbook by the academic A.Pogorelov.

Courseware in geometry "PMC DG"

Courseware "PMC DG" consists of the following components.

- The original dynamic geometry package DG (scientific adviser – Sergey Rakov, programmer – Cyril Osenkov, mathematical support – Victor Gorokh).
- DGF - Library – library of about 1000 of dynamic drawings devoted to different math courses from primary school level up to the university level and research problems.
- Guidebook for teachers "Discovering geometry through the computer experiments in DG environment" – developed by 9 authors, including teachers of schools and scientists, programmers and discussing variety of problems - methodological, technological, pedagogical etc. aspects concerning the implementation of ICT (in particular DG) in geometry course.
- Guidebook for students "Discovering geometry with DG" (7^{th} – 9^{th} grade) - systematical discussions of beneficial use of DG in geometry which step by step follows the classic geometry course in new way – way of constructivism, modeling, experiments, explorations. Guidebook is the electronic course with implemented screen copies of correspondent dynamic drawings, which "revive" by the mouse click, loading DG with loaded correspondent dynamic drawing.
- "Explorations in geometry" – guidebook for students which consists of worksheets for learning explorations (2 explorations for each of 6 semesters).

1500 copies of DG will be shared among the schools involved in a project "Pilot School – 2000/Computarisation of a rural school" in shortest time. It is a great problem now to prepare teachers for its usage. The special courses "Courseware DG" for the pre-service and in-service teachers do not solve all the problems arising before teachers. Our next project is an educational site in Geometry for teachers and students. We give below some excerptions from the student guide as illustrations of a way for transformation of a classic geometric course in constructive way.

Pythagorean and Egypt triangles

Pythagorean theorem tells us that in rectangular triangle the square of the hypotenuse length equals to the sum of squares of its cathetuses lengths. Does it right the inverse: if in a triangle the square of one side length equals to the sum of squares of the lengths the two other sides then the triangle is rectangular triangle and its first side is its hypotenuse? This is really right assertion and it follows from the discussed above in the textbook solution of the problem. The inverse to the Pythagorean theorem yields methods for constructing right angle: we can use for that the circle rope with knots which divide the rope pieces in given ratio, the components of which satisfy the equation $c^2 = a^2 + b^2$, for example 5:4:3.

Problem

Find the method for constructing the right angle with a rope with knots. Does the triangle with the ratio of peaces 5:4:3 is the best one for that? Why do or why not?

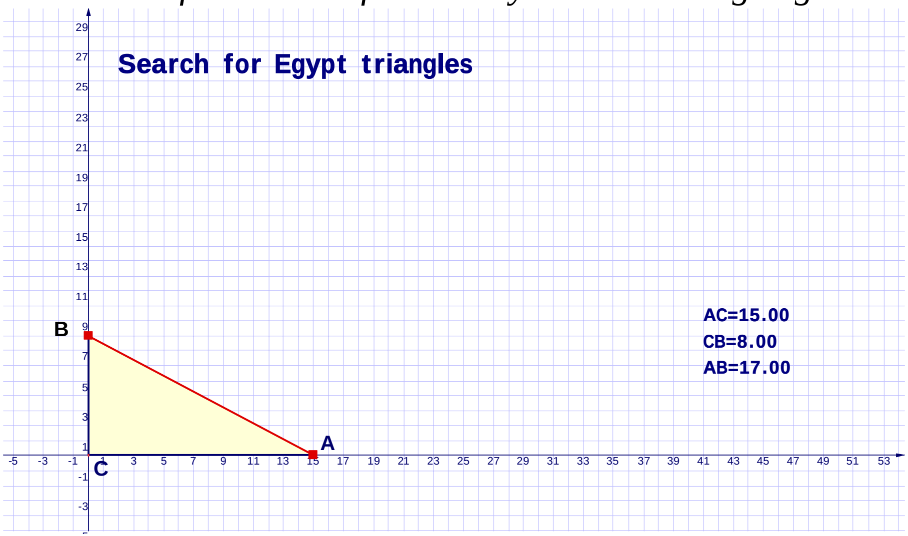
The rectangular triangle with ratio of the side lengths 5:4:3 is called the **Egypt Triangle**.

Do the other rectangular triangles with the natural ratio of the side lengths exist (naturally, except of the triangles with the side lengths multiples to the lengths of the sides of the Egypt triangle, such as 6 meters, 8 meters and 10 meters)? Such a triangles are called the **Pythagorean Triangles**. Those for whom this question is interesting can search the literature on the Defiant equations (such name have equations, for which the integer solutions are searched). The equation $x^2 + y^2 = z^2$ is an example of the Defiant equation, if we are looking for the integer solutions.

You can search for the other integer solutions for this equation in DG environment .

Problem

Construct a dynamic drawing in DG environment for automatical (not automatic) search for the Pythagorean Triangles (integer solutions of the equation $x^2 + y^2 = z^2$). A variant of the screen for such dynamic drawing is given below.



Pic.1 DGF "Pythagorean Triangles"

At the picture the candidate for the Pythagorean Triangle is given. You should check by exact calculation that given triangle is really Pythagorean Triangle. *How do you think, why such checking you should make?*

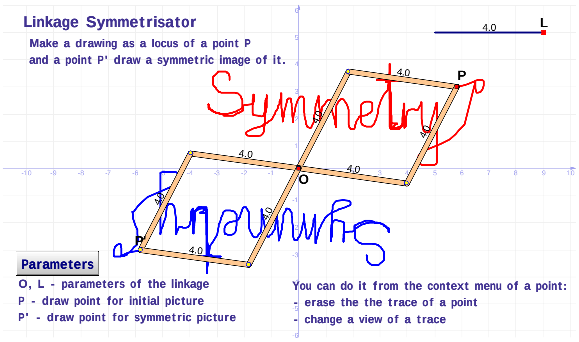
Problem

1. *Find as much as possible of Pythagorean Triangles with the help of constructed dynamic drawing*[2].
2. *Propose the common method for searching the solutions of the Defiant equation with two variables in DG environment.*
3. *Propose the method for searching the solutions of Defiant equations with three variables in DG environment in some particular cases which generalize the case* $x^2 + y^2 = z^2$.

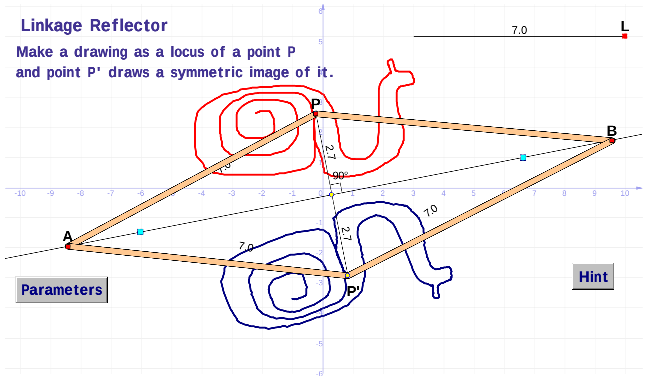
[2] If drag the mouse with pressed key *Shift* it changes its place in discrete way over the knots of the coordinate net.

Geometric Transformations, DG and Linkages

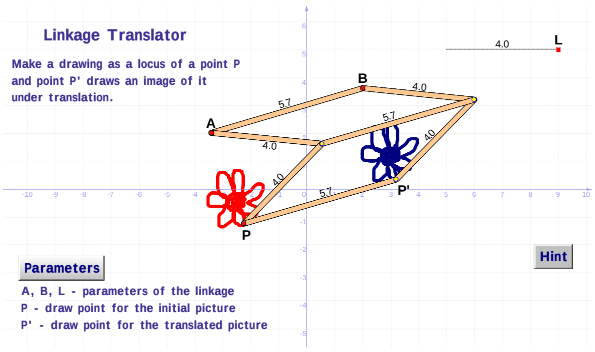
It is a very interesting and productive work to construct linkages, play with them and explore properties of geometric transformations with appropriate electronic linkage. The following copies of the screens illustrate constructions and activities in DG environment at these topics.



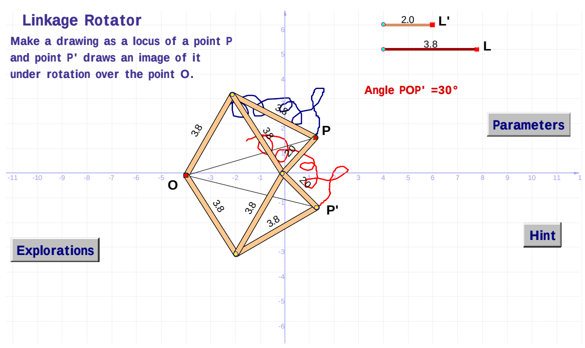
Pic. 1 Linkage Symmetrisator



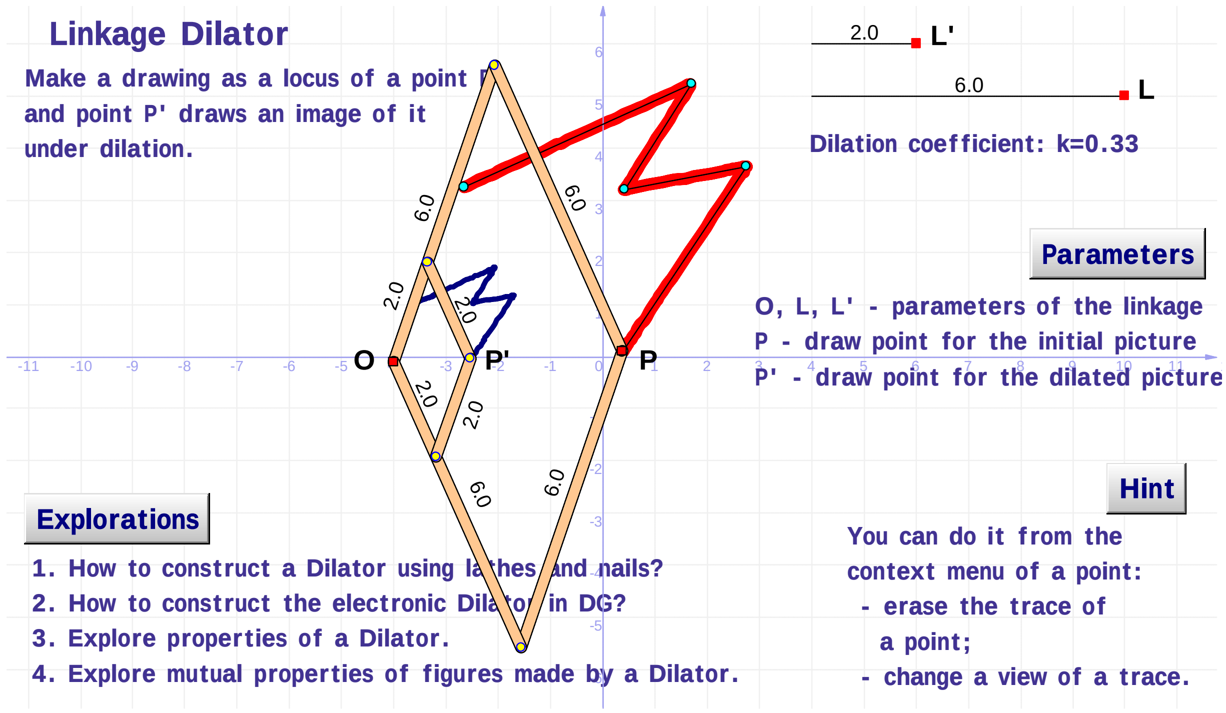
Pic. 2 Linkage Reflector



Pic. 3 Linkage Translator



Pic. 4 Linkage Rotator



Pic. 5 Linkage Dilator

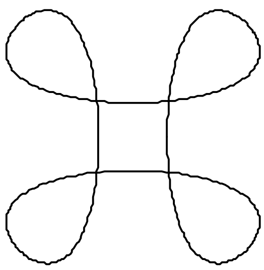
The DG – realization of geometric primitives and transformation offer the user to construct simpler but more powerful “electronic linkages”. Such possibilities are in the main the result of one internal property of a point at the DG - segment to divide the segment at the invariant ratio under the transformations of a segment. Such property inspired a special type of electronic linkages, which can be called the “gum linkages” (because of the mentioned above property of an “electronic segment” can be modeled with homogeneous gum strip or gum stick). For example the simplest “Gum Dilator ” is a segment with a point at them. It is interesting that “Gum Tools” can have the entire plane as a domain[3] (obviously, real tools have bounded domains).

Problem

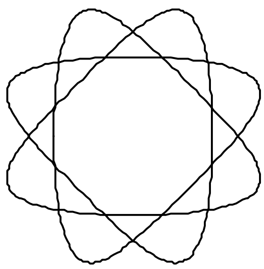
1. *Construct the possibly simplest “Gum Symmetrisator”, “Gum Reflector”, “Gum Translator”, “Gum Rotator”, “Gum Dilator” with entire plane as a domain.*
2. *Find as much as possible real life problems, which can be solved approximately with the help of linkages.*
3. *Construct your own linkage and explore its properties.*

Magic curves and DG

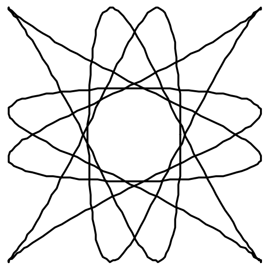
All the curves at the pictures 1-6 are built with the same dynamic drawing (DD), moreover this DD has only one parameter. Really magic variety and magic beauty. To explore this variety and beauty, to understand their background are interesting topics for variety of projects.



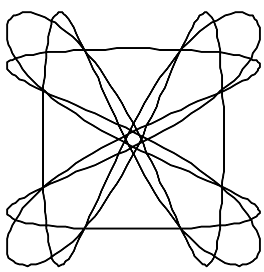
Pic.6



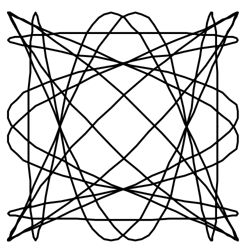
Pic. 7



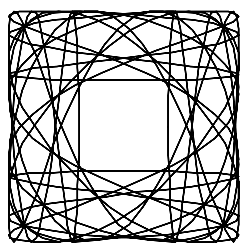
Pic. 8



Pic. 9



Pic. 10

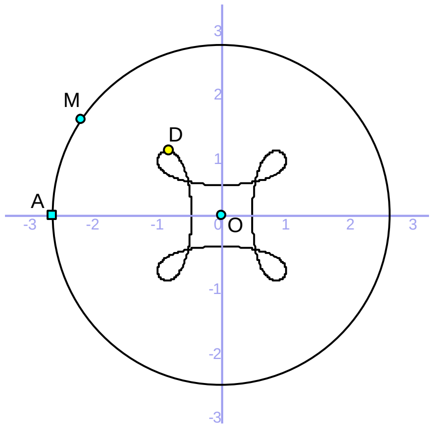


Pic.11

[3] The domain of the linkage is a set of points, the correspondent images of which can be constructed with this linkage.

This is a description of correspondent dynamic drawing constructing.

- Build a point *O* – the origin of the coordinate system (command **Figures\Analytic\Point)**.
- Build a point A at the abscise coordinate axe (tool **Point on figure**).
- Build a circle with the center *O*, which pass through the point *A* (tool **Circle**).
- Build a point *M* at the circle (tool **Point on figure**).
- Build a point *D*, with coordinates analytically expressed through the coordinates of a point *M* in such a way: $x_D = \sin(x_M)$, $y_D = \sin(y_M)$ (command **Figures\Analytic\Point)**.
- Build the dynamic locus of a point *D* when point M runs over the circle (tool **Dynamic locus**).



Pic. 12

The result is shown at the picture 12. By dragging the point *A*, we can receive all the mentioned above variety of curves and a lot of other ones. By the tool **Point properties** we can easy find parametric equations of these curves**.**

Problem

1. *Construct the dynamic drawing for the sketch of the 6 – parametric curve given by the system of equations:* $\begin{cases} x(t) = A_1 \sin(k_1 t + m_1) \\ y(t) = A_2 \sin(k_2 t + m_2) \end{cases}$, $t \in [0, T]$.
2. *Explore, how the sketch of a curve depends from its parameters.*
3. *Propose real life processes, which can be described by such analytic and graphical model.*

Problem

1. *Construct the dynamic drawing for the sketch of the 4 – parametric family of curves given by the system of equations:* $\begin{cases} x(t) = v_1 t \\ y(t) = -gt^2 + v_2 t + h_0 \end{cases}$, $t \in [0, T]$.
2. *Explore, how the sketch of a graph depends from its parameters.*
3. *Propose real life process, which can be described by such analytic and graphical model.*

Literature

1. A.Pogorelov, « Geometry», textbook for students of the 7th – 9th grade, Kijv, «Osvita», 1999, 224p.
2. S.Rakov, V.Gorokh, « Discovering geometry with DG», guidebook for students of the 7th – 9th grade, 2002, electronic course.